THE PERT NETWORK DIAGRAM

PERT (Program Evaluation Review Technique) network diagram is used to model a project's activities and their relationships as a network. It was developed to take account of the uncertainty surrounding estimates of task durations. The difference between the CPM and the PERT methods is that the CPM uses a single estimate for the duration of each task, whereas the PERT method uses 3 estimates. These 3 estimates include:

- **Optimistic (a)**
- **Most Likely (m)**
- **Pessimistic (b)**

These 3 estimates are combined to calculate the t_e values. The formula is depicted below:

$$te = \frac{a + 4m + b}{6}$$

After the t_e values are calculated, the standard deviation **(s)** can be calculated using the following formula:

$$s = \frac{b - a}{6}$$

The next step is to calculate the **z - value**. The formula to calculate the z value is depicted below:

$$z = \frac{T - te}{s}$$

Remember that the **T value is the target date** of the specific project and the **z value is calculated on the last activity.**

The PERT event labelling convention adopted here indicates event number and its target date along with the calculated values for expected time and standard deviation.
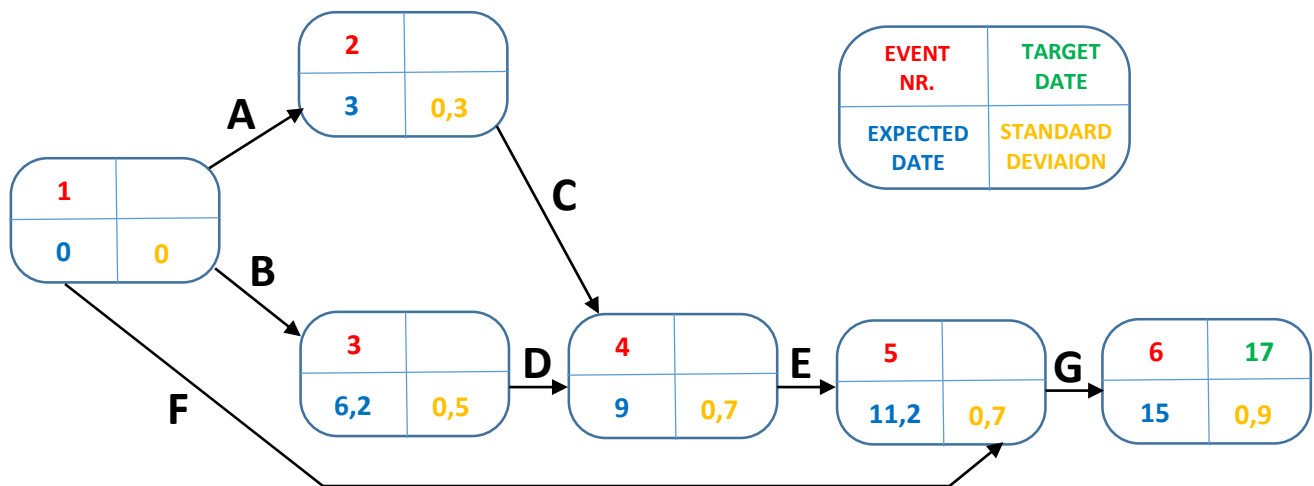
EVENT NR.	TARGET DATE
EXPECTED DATE	STANDARD DEVIAION
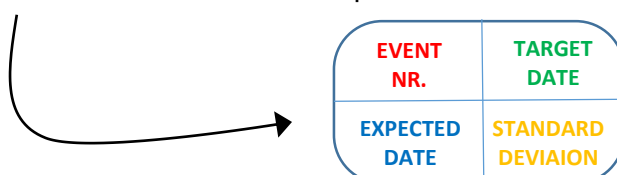
NB: KNOW YOUR FORMULAS BY HEART. IT WILL NOT BE GIVEN TO YOU IN THE EXAM.

EXAMPLE 1:

In the table below the optimistic, most likely and pessimistic values are shown. The table also indicates the expected and standard deviation. **The Target date is 17.**

	OPTIMISTIC (a)	MOST LIKELY (m)	PESSIMISTIC (b)	EXPECTED (t_e)	STANDARD DEVIATION (S)
A	2	3	4	3	0,3
B	5	6	8	6,2	0,5
C	3	4	5	4	0,3
D	1	3	4	2,8	0,5
E	2	2	3	2,2	0,2
F	1	3	5	3	0,7
G	2	4	5	3,8	0,5

The t_e and S values are indicated in the figure above.

PLEASE NOTE: Standard deviation for the project events can be calculated by carrying out a forward pass using the activity standard deviations in a manner similar to that used with expected duration.

There is, however, one small difference – to add two standard deviations we must add their squares and then find the square root of the sum.

For example: Calculating the S value of activity 4.
- Path B+D: $\sqrt{(0,5)^2 + (0,5)^2} = 0,7$ and
- Path A+C: $\sqrt{(0,3)^2 + (0,3)^2} = 0.4$

Path B+D has the greater answer. Therefore the s value of **activity 4** is **0.7.**

Calculating the S value of activity 5.
- Path (B+D) + E = $\sqrt{(0,7)^2 + (0,2)^2} = 0.7$ and
- Path F = 0.7 (from the table).

The answers are equal, therefore the s value of **activity 5** is **0.7.**

Calculating the z value:

$$Z = \frac{(T - te)}{S} = \frac{17 - 15}{0,9} = 2.2$$

According to Fig. 7.8 on page 181 of the textbook, the probability of not meeting the target date is less than 10%

EXAMPLE 2: (attempt the example without looking at the solutions below)

In the PERT network illustrated in the figure below, the target date for the completion of the project is **11 weeks**.
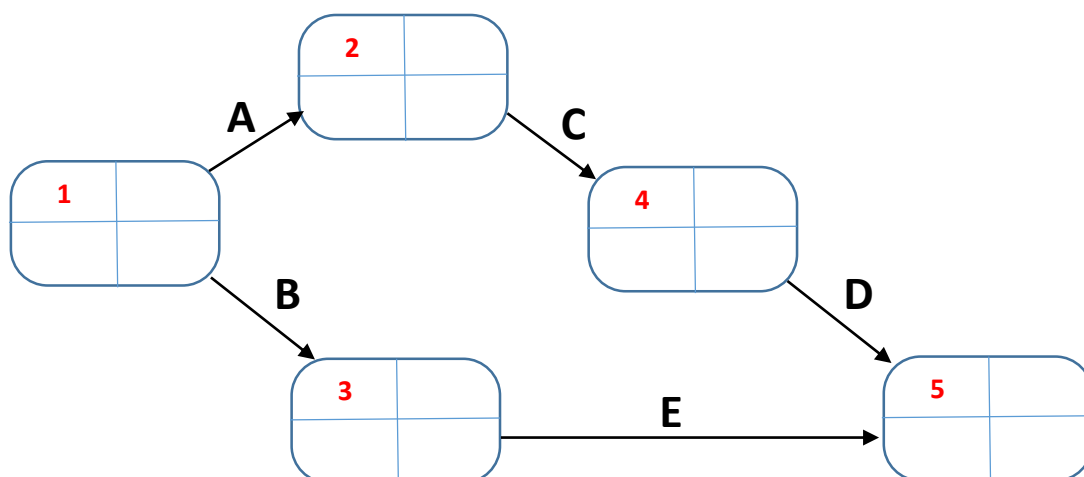

EVENT NR.	TARGET DATE
EXPECTED DATE	STANDARD DEVIAION
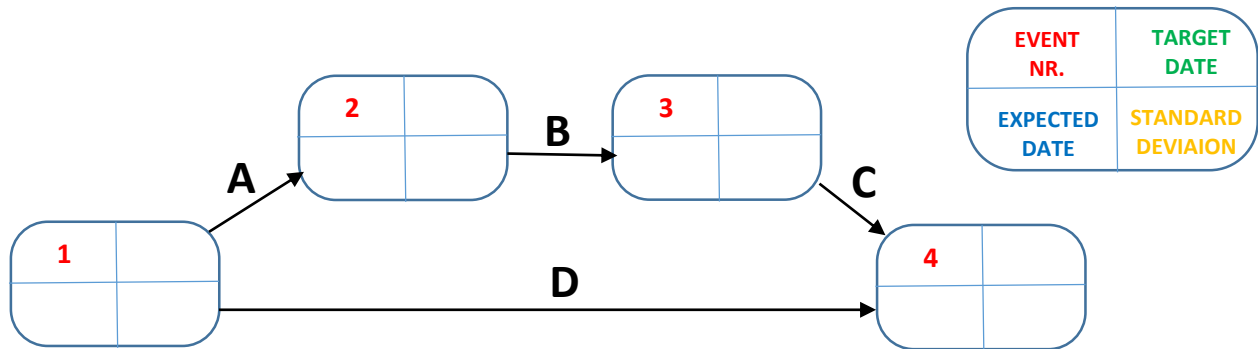
	OPTIMISTIC (a)	MOST LIKELY (m)	PESSIMISTIC (b)	EXPECTED (t_e)	STANDARD DEVIATION (s)
A	3	4	5		
B	1	2	3		
C	2	3	4		
D	4	5	6		

Use the table above to calculate the following:

2.1   Calculate the **Expected (t_e)** values and **Standard Deviation (s)** for all the activities and the events and indicate all the **(t_e)** and **(s)** values on the diagram.

2.2   Calculate the Z value of the last event.

2.3   According to Figure 7.8 (p.181) in your textbook, what is the probability of not meeting the target date?

EXAMPLE 3: (attempt the example without looking at the solutions below)

In the PERT network illustrated in the figure below, the targeted date for the completion of the project is nine (9) weeks.

PERT NETWORK

	OPTIMISTIC (a)	MOST LIKELY (m)	PESSIMISTIC (b)	EXPECTED (t$_e$)	STANDARD DEVIATION (s)
A	1	2	3		
B	3	4	5		
C	2	3	4		
D	1	2	3		
E	3	4	5		

Use the table above to calculate the following:
3.1   Calculate the **expected (t$_e$)** values and **standard deviation (s)**
3.2   Indicate the **t$_e$** and **s** values on the diagram.
3.3   Calculate the Z value on the last event
3.4   According to figure 7.5 in your textbook, what is the probability of not meeting the target date?

SOLUTION of EXAMPLE 2:
2.1

Use the following formula for t$_e$ values:

$$t_e = \frac{(a+4m+b)}{6}$$

t$_e$ value for activity A: $\frac{(3+4(4)+5)}{6} = \frac{24}{6} = \mathbf{4}$

t$_e$ value for activity B: $\frac{(1+4(2)+3)}{6} = \frac{12}{6} = \mathbf{2}$

t$_e$ value for activity C: $\frac{(2+4(3)+4)}{6} = \frac{18}{6} = \mathbf{3}$

t$_e$ value for activity D: $\frac{(4+4(5)+6)}{6} = \frac{30}{6} = \mathbf{5}$

Use the following formula for s values:
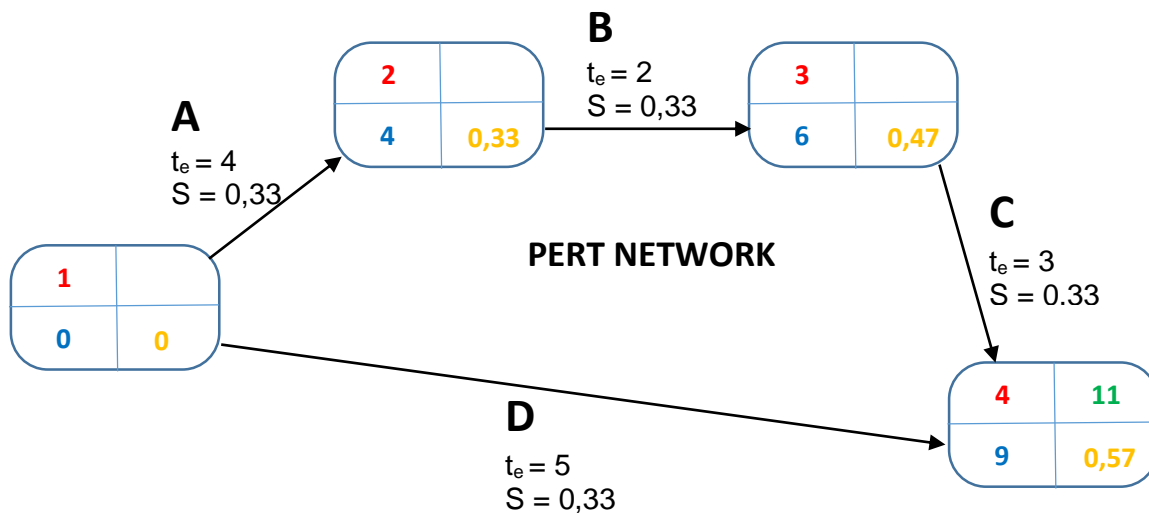
$$S = \frac{(b-a)}{6}$$

s value for activity A: $\frac{(5-3)}{6} = \frac{2}{6} = \frac{1}{3} = \mathbf{0.33}$

s value for activity B: $\frac{(3-1)}{6} = \frac{2}{6} = \frac{1}{3} = \mathbf{0.33}$

s value for activity C: $\frac{(4-2)}{6} = \frac{2}{6} = \frac{1}{3} = \mathbf{0.33}$

s value for activity D: $\frac{(6-4)}{6} = \frac{2}{6} = \frac{1}{3} = \mathbf{0.33}$

	OPTIMISTIC (a)	MOST LIKELY (m)	PESSIMISTIC (b)	EXPECTED (t$_e$)	STANDARD DEVIATION (s)
A	3	4	5	4	0,33
B	1	2	3	2	0,33
C	2	3	4	3	0,33
D	4	5	6	5	0,33

PERT NETWORK

Standard deviation of event 4 is calculated as follows:
Two paths are possible i.e. **A-B-C** or **D**.

The standard deviation selected will be the greater of the standard deviations of activities
- Standard deviation of A-B-C:

$$S = \sqrt{A^2 + B^2 + C^2} = \sqrt{0,33^2 + 0,33^2 + 0,33^2} = 0.57$$

- or the standard deviation of activity D = 0.33

Therefore, the standard deviation for event 4 is **0.57.**

2.2 $\quad Z = \dfrac{(T - te)}{S}$

$$= \frac{(11 - 9)}{0,57} = \frac{2}{0,57}$$

$$= 3.51$$

2.3 $\quad$ Since the Z value is 3.51, the probability of not meeting the target date is 0 (zero). Refer to figure 7.8 in your textbook.

SOLUTION of EXAMPLE 3:

3.1 $\quad$ **Use the following formula for t_e values:**

$$t_e = \frac{(a+4m+b)}{6}$$

t_e value for activity A: $\quad \dfrac{(1+4(2)+3)}{6} = \dfrac{12}{6} = 2$

t_e value for activity B: $\quad \dfrac{(3+4(4)+5)}{6} = \dfrac{24}{6} = 4$

t_e value for activity C: $\quad \dfrac{(2+4(3)+4)}{6} = \dfrac{18}{6} = 3$

t_e value for activity D: $\quad \dfrac{(1+4(2)+3)}{6} = \dfrac{12}{6} = 2$

t_e value for activity E: $\quad \dfrac{(3+4(4)+5)}{6} = \dfrac{24}{6} = 4$

Use the following formula for s values:

$$S = \frac{(b-a)}{6}$$

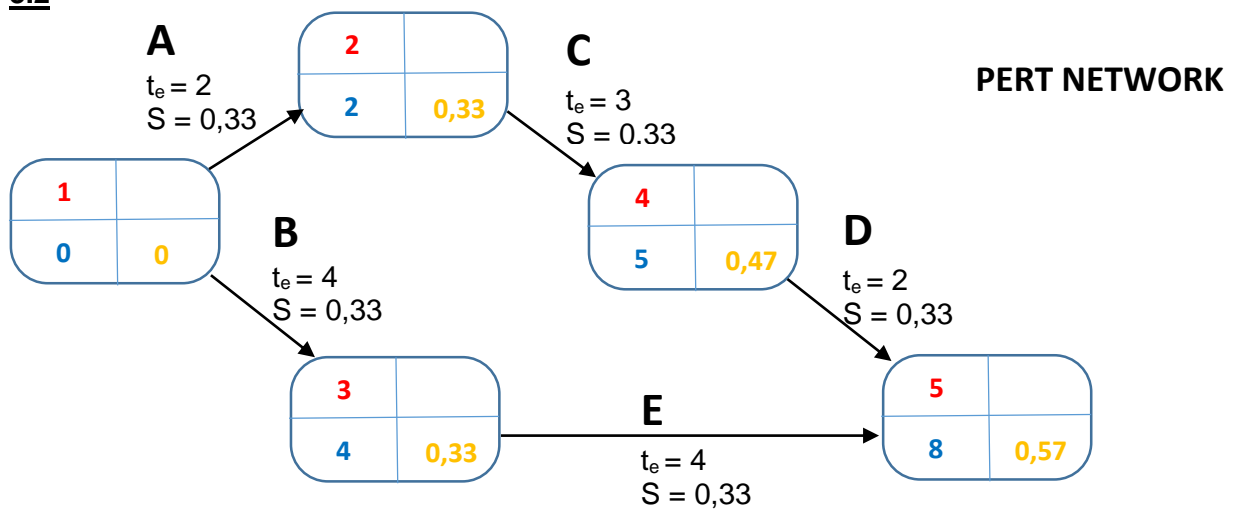*s value for activity A:* $\quad \frac{(3-1)}{6} = \frac{2}{6} = \frac{1}{3} = \textbf{\textit{0.33}}$

s value for activity B: $\quad \frac{(5-3)}{6} = \frac{2}{6} = \frac{1}{3} = \textbf{\textit{0.33}}$

s value for activity C: $\quad \frac{(4-2)}{6} = \frac{2}{6} = \frac{1}{3} = \textbf{\textit{0.33}}$

s value for activity D: $\quad \frac{(3-1)}{6} = \frac{2}{6} = \frac{1}{3} = \textbf{\textit{0.33}}$

s value for activity E: $\quad \frac{(5-3)}{6} = \frac{2}{6} = \frac{1}{3} = \textbf{\textit{0.33}}$

3.2

A $t_e = 2$ $S = 0,33$

B $t_e = 4$ $S = 0,33$

C $t_e = 3$ $S = 0.33$

D $t_e = 2$ $S = 0,33$

E $t_e = 4$ $S = 0,33$

PERT NETWORK

3.3 Calculate the Z value on the last event

$$Z = \frac{(T - t_e)}{S}$$

$$= \frac{(9 - 8)}{0,57} = \frac{1}{0,57} = \textbf{1,75}$$

3.4 The probability of not meeting the target date is approximately 5%. Any value from 4% to 6% is acceptable because it is not easy to pinpoint the precise value from the graph.