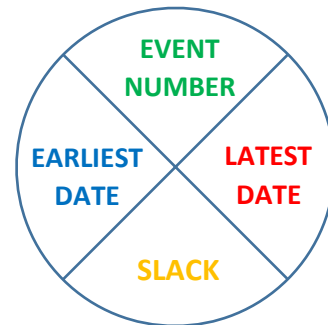


# DRAWING A CPM DIAGRAM (ACTIVITY-ON-ARROW)

## EXAMPLE 1:

Consider the list of tasks with dependencies and estimated durations (in weeks) in the table below:

Task	Precedents	Duration (weeks)
A	None	1
B	A	2
C	B	2
D	A	5
E	C, D	5
F	E	2
G	A	4
H	G	4
I	F, H	1
J	D, I	2



1. Draw a CPM network for the list of tasks in the table above to illustrate the interaction of activities.
2. Identify the critical path and its duration.
3. After carefully considering the problem again, the project team decides that an alternative to the original plan would be to extend the duration of activity C to four weeks. This would allow activity E to drop to three weeks. Identify what the resulting critical activities are. Do these changes affect the total duration of the project?

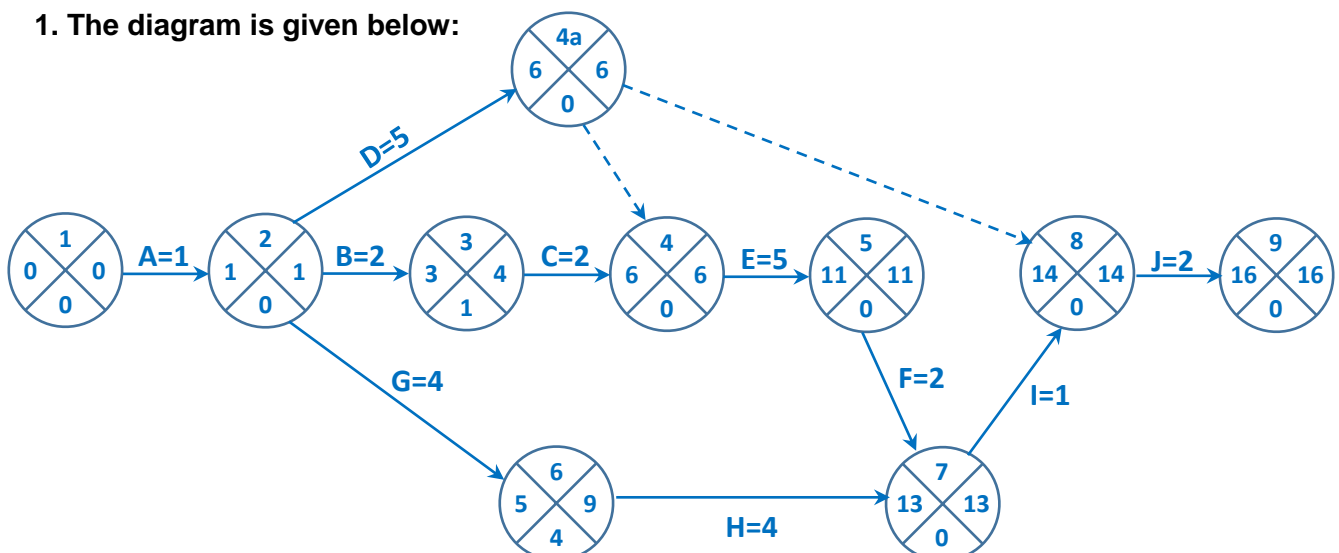
### A few aspects to bear in mind when drawing a CPM network:

- make sure that you understand and can apply the rules on pages 128-131 of the textbook;
- the **forward pass** calculates the **earliest date** each event may start and the earliest date on which it may be completed;
- the **backward pass** calculates the **latest date** when each event may be started and the latest date when it can be achieved or finished;
- **event dates** are recorded on the diagram and activity dates on the activity table.

In this example both processes D and C have to be finished and serve as input to process E, but process D and I also serve as input to process J. Thus a dummy activity was created, namely process 4a, to illustrate the duality.

## SOLUTION:

1. The diagram is given below:



2. In order to determine the critical path, we first of all list all of the different paths:

path 1 A, D, E, F, I, J = 16 weeks

path 2 A, B, C, E, F, I, J = 15 weeks

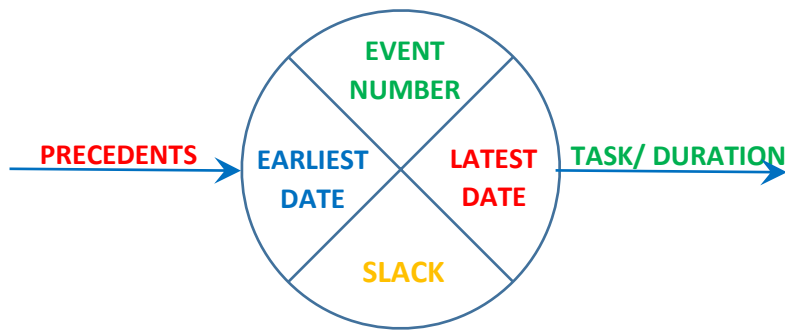
path 3 A, G, H, I, J = 12 weeks

path 4 A, D, J = 8 weeks

**Path 1 is thus the critical path.**

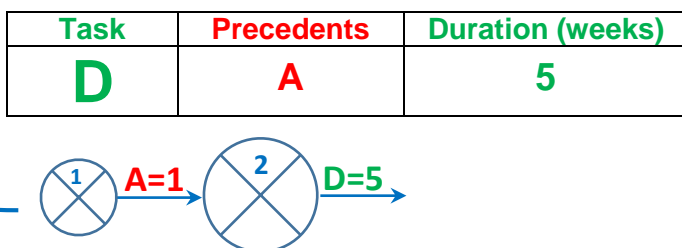
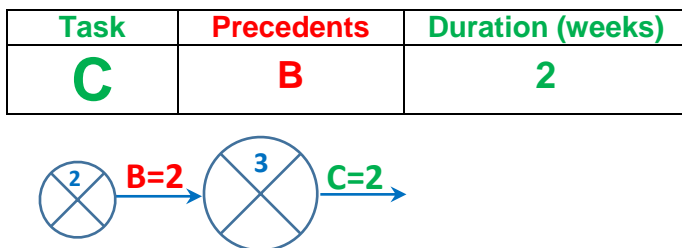
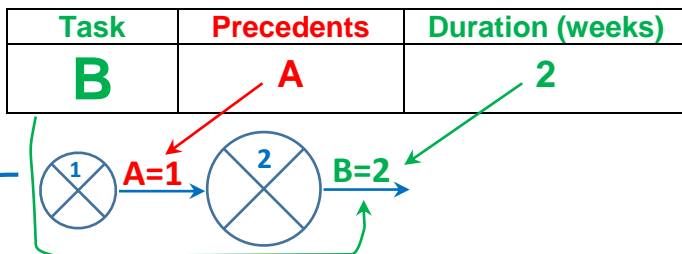
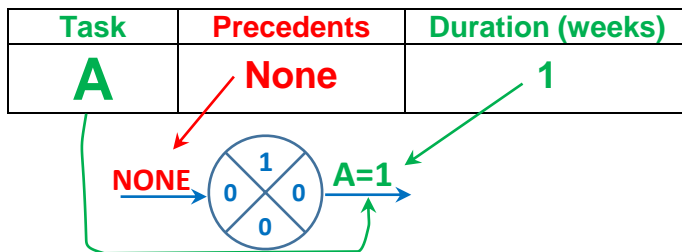
3. Path 1 drops to 14 weeks and path 2 to 15 weeks. Now the critical path will be path 2: A, B, C, E, F, I, J. The minimum duration drops to 15 weeks.

# DETAILED EXPLANATION OF EXAMPLE 1



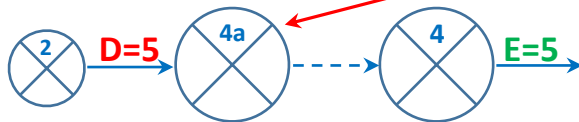
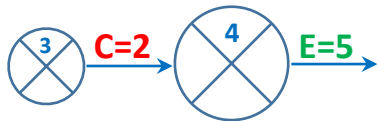
## STEP 1:

Draw each node/event and link it to each other at the end to illustrate the interaction of activities:



The **EVENT NUMBER** IS THE SAME BECAUSE ITS PRECEDENT IN BOTH CASES ARE **A**

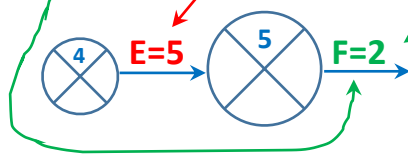
Task	Precedents	Duration (weeks)
<b>E</b>	<b>C, D</b>	<b>5</b>



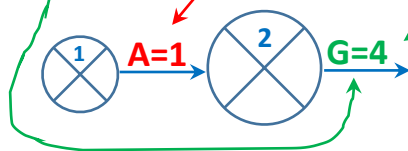
In this example both activity C and D have to be finished and serve as input to activity E, but activity D and I also serve as input to activity J (see below)  
Thus a dummy activity was created, namely Event 4a, to illustrate the duality.

### CREATING A DUMMY ACTIVITY

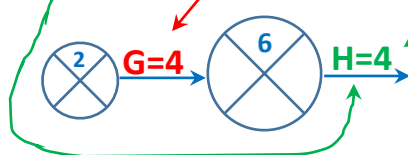
Task	Precedents	Duration (weeks)
<b>F</b>	<b>E</b>	<b>2</b>



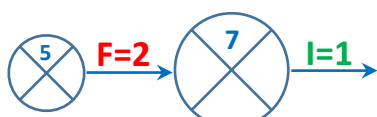
Task	Precedents	Duration (weeks)
<b>G</b>	<b>A</b>	<b>4</b>



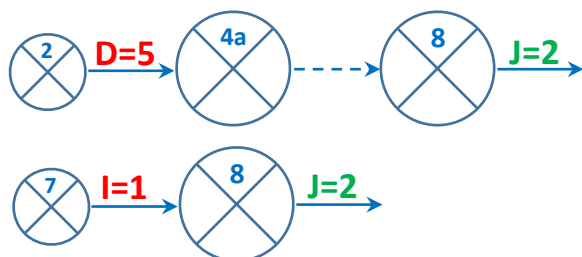
Task	Precedents	Duration (weeks)
<b>H</b>	<b>G</b>	<b>4</b>



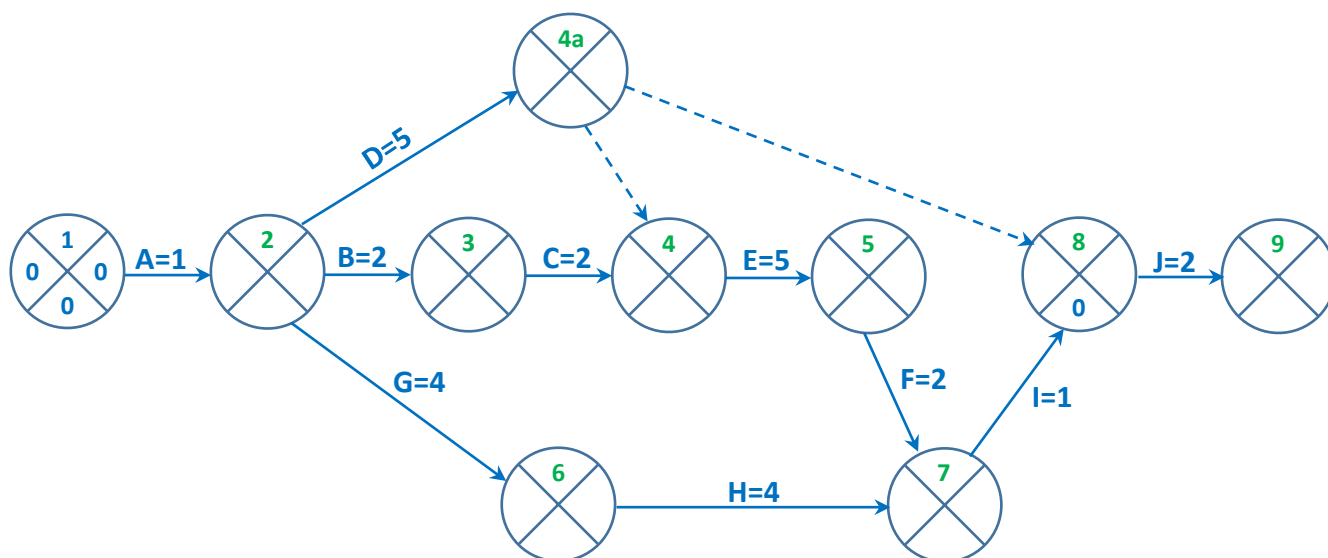
Task	Precedents	Duration (weeks)
<b>I</b>	<b>F, H</b>	<b>1</b>



Task	Precedents	Duration (weeks)
<b>J</b>	<b>D, I</b>	<b>2</b>



LINKING ALL THE NODES (or EVENTS) ACCORDING TO THE TASKS SHOULD GIVE YOU THE FOLLOWING **CPM NETWORK STRUCTURE** to ILLUSTRATE THE INTERACTION OF ACTIVITIES:



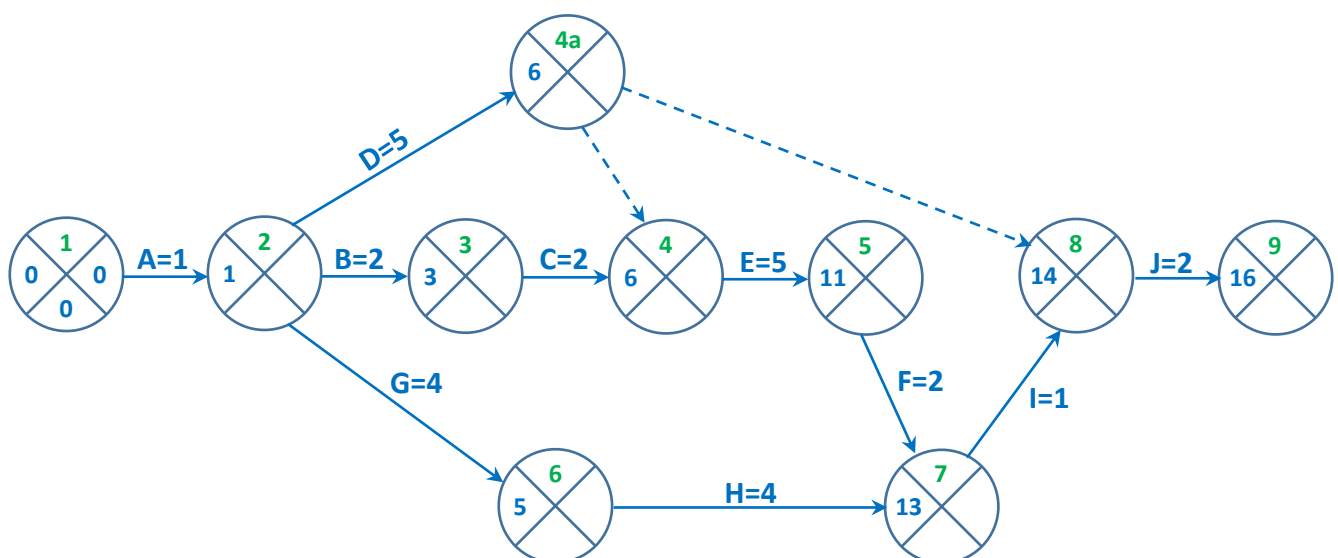
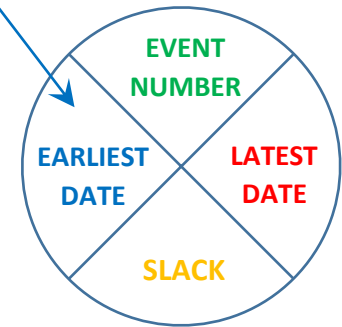
# STEP 2:

## CALCULATING THE EARLIEST DATE (FORWARD PASS)

- Take note of the Precedents of each NODE/EVENT (as shown below)

Event 4, 7 and 8 has TWO precedents (The larger number should be taken as the Earliest Start Date)

EVENT NR	CALCULATIONS: EARLIEST DATE + DURATION	EARLIEST DATE
1	Earliest Date of Event 1 $= 0$	0
2	Earliest Date of Event 1 + A's Duration $0 + 1 = 1$	1
3	Earliest Date of Event 2 + B's Duration $1 + 2 = 3$	3
4a	Earliest Date of Event 2 + D's Duration $1 + 5 = 6$	6
4	Earliest Date of Event 3 + C's Duration $3 + 2 = 5$  Earliest Date of Event 4a + Dummy Duration $6 + 0 = 6$	6
5	Earliest Date of Event 4 + E's Duration $6 + 5 = 11$	11
6	Earliest Date of Event 1 + G's Duration $1 + 4 = 5$	5
7	Earliest Date of Event 5 + F's Duration $11 + 2 = 13$  Earliest Date of Event 6 + H's Duration $5 + 4 = 9$	13
8	Earliest Date of Event 4a + Dummy Duration $6 + 0 = 6$  Earliest Date of Event 7 + I's Duration $13 + 1 = 14$	14
9	Earliest Date of Event 8 + J's Duration $14 + 2 = 16$	16

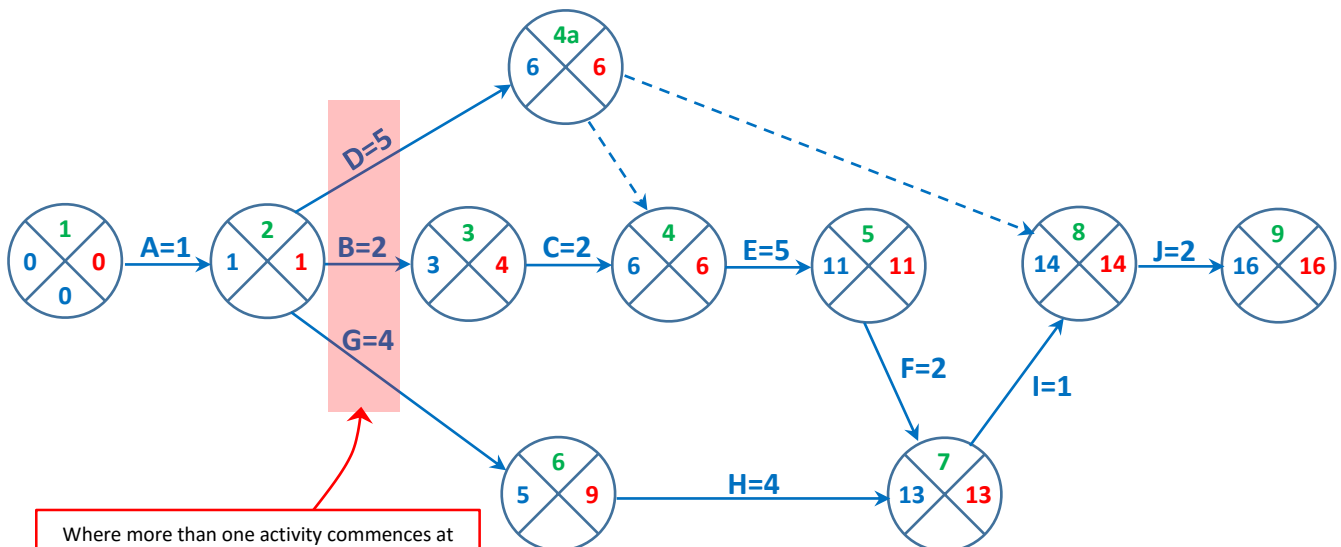
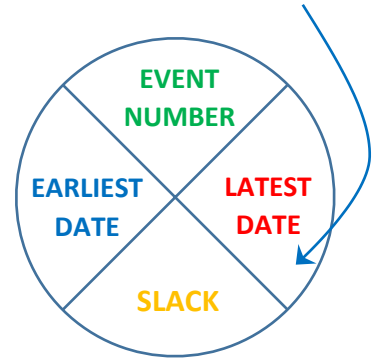


# STEP 3:

## CALCULATING THE LATEST DATE (BACKWARD PASS)

- Take note of the **Precedents** of each **NODE/EVENT** (as shown below)
- If you do the **BACKWARD pass**, try to start from the last event and work your way to the front.
- Look at Event nr. 2 that has more than 1 activity commencing from it.

EVENT NR	CALCULATIONS: (LATEST DATE – DURATION)	LATEST DATE
9	Latest Date of the last event is assumed to be the same the <b>Earliest</b> Date	16
8	Latest Date of Event 9 – J's Duration 16 - 2	14
7	Latest Date of Event 8 - I's Duration 14 - 1	13
6	Latest Date of Event 7 - H's Duration 13 - 4	9
5	Latest Date of Event 7 - F's Duration 13 - 2	11
4	Latest Date of Event 5 - E's Duration 11 - 5	6
4a	Latest Date of Event 4 - <b>Dummy Duration</b> 6 - 0	6
3	Latest Date of Event 4 - C's Duration 6 - 2	4
2	Latest Date of Event 4a - D's Duration 6 - 5	1
1	Latest Date of Event 1 Start with 0	0



Where more than one activity commences at a common event (as in **event 2** above), we take the latest date of event 3, 6 and 4a minus their respective durations and select the **lowest latest date** of the three to calculate the latest date of **Event 2**.

**FOR EXAMPLE:**

Event 3: (4-2=2)

Event 4a: (6-5=1)

Event 6: (9-4=5)

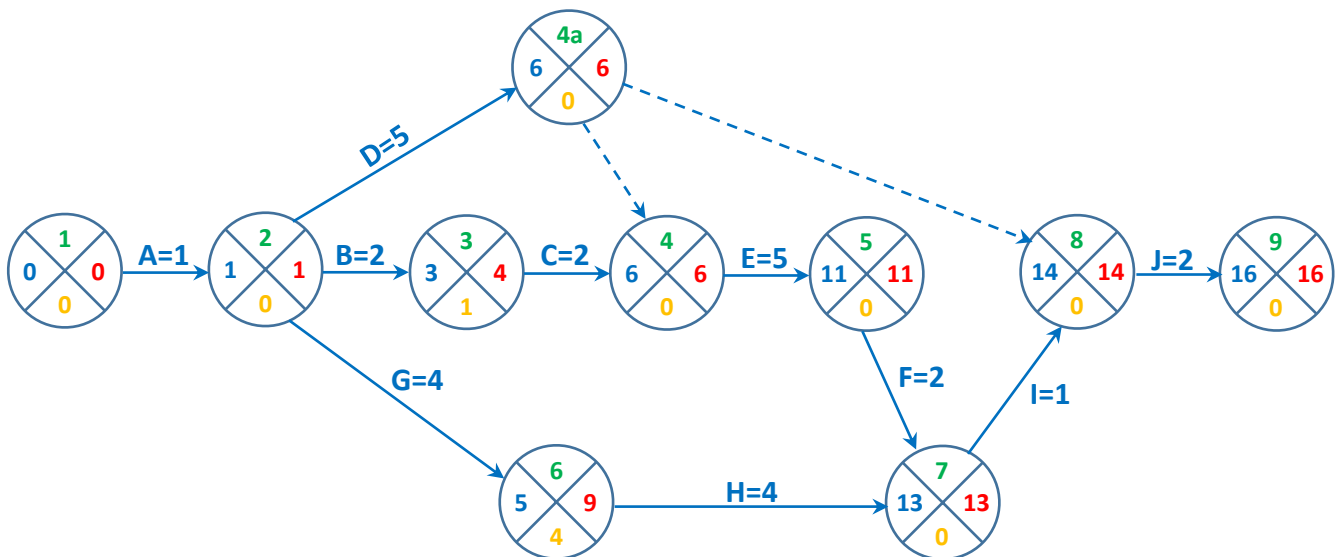
Therefore, the latest date of event 2 = **1**

# STEP 4:

## CALCULATING SLACK

- Latest Date minus Earliest Date

EVENT NR	CALCULATIONS: (LATEST DATE – EARLIEST DATE)	SLACK
1	Latest Date of Event 1 Start with 0	0
2	Latest Date – Earliest Date of Event 2 $1 - 1 = 0$	0
3	Latest Date – Earliest Date of Event 3 $4 - 3 = 1$	1
4a	Latest Date – Earliest Date of Event 4a $6 - 6 = 0$	0
4	Latest Date – Earliest Date of Event 4 $6 - 6 = 0$	0
5	Latest Date – Earliest Date of Event 5 $11 - 11 = 0$	0
6	Latest Date – Earliest Date of Event 6 $9 - 5 = 0$	4
7	Latest Date – Earliest Date of Event 7 $13 - 13 = 0$	0
8	Latest Date – Earliest Date of Event 8 $14 - 14 = 0$	0
9	Latest Date – Earliest Date of Event 9 $16 - 16 = 0$	0





## EXAMPLE 2:

1. Consider the following list of tasks with dependencies and estimated durations reflected in the table. Draw a CPM network (activity-on-arrow diagram) to illustrate the interaction of activities. Include detailed information on the nodes.

Task	Precedents	Duration (weeks)
A	None	6
B	None	5
C	B	12
D	B	9
E	A	28
F	A	7
G	D	11
H	F, G	6

2. Write down the critical path using the letters of the tasks and calculate and write down the duration of the project. Write down the other paths and their durations.

## EXAMPLE 3:

1. There are a number of different conventions that have been adopted for entering information on network and activity planning diagrams, such as activity-on-arrow networks and the Precedence or activity-on-node network diagrams. Give the naming convention of events for activity-on-arrow networks as used in Hughes & Cotterel. Use clearly well labelled diagrams.
2. Consider the following activities with their precedents and durations. Draw a complete Activity-on-arrow network diagram. Include the event number, earliest date, latest date and slack on each node by completing both a forward and backward pass. Clearly indicate the total duration and critical path.

Task	Precedents	Duration (weeks)
A	None	34
B	A	20
C	A	15
D	C	25
E	B	12
F	D, E	7
G	D, E	6
H	F	30
I	G	28
J	I, H	6

## EXAMPLE 4:

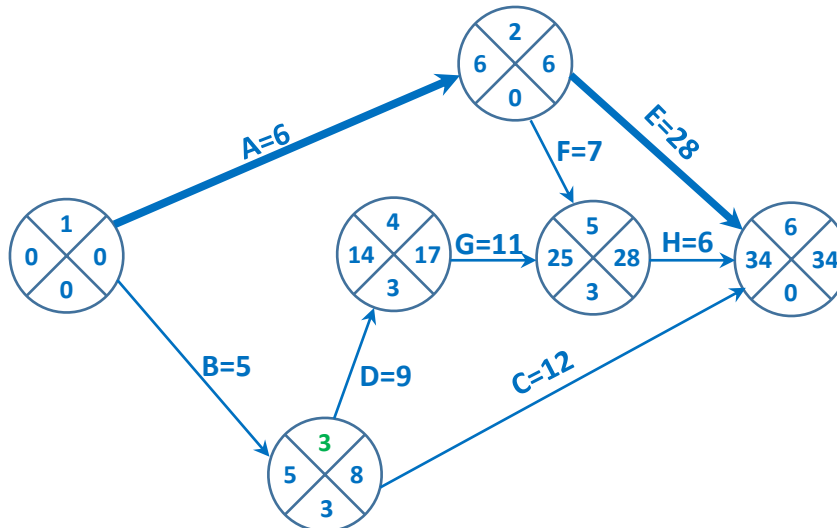
1. Consider the following list of tasks with dependencies and estimated durations reflected in table 1. Draw a CPM network (activity-on-arrow diagram) to illustrate the interaction of activities. Include all the values in the nodes. Indicate the critical path on the diagram

Task	Precedents	Duration (weeks)
A	None	5
B	None	9
C	None	11
D	A	8
E	B	5
F	B	12
G	C	10
H	G	5
I	D, E	11
J	F, H	4
K	G	4

2. Write down the critical path(s) using the letters of the tasks and calculate and write down the duration of the project. How many paths are there in total? Identify them all and write them down.
3. What will the effect on the project be if the duration of activity A changes to 12 weeks

## SOLUTION of EXAMPLE 2:

1.

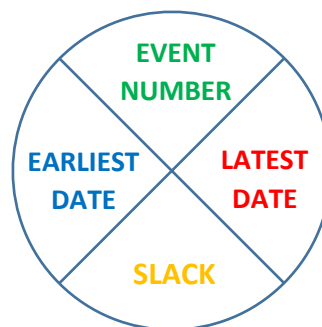


2.

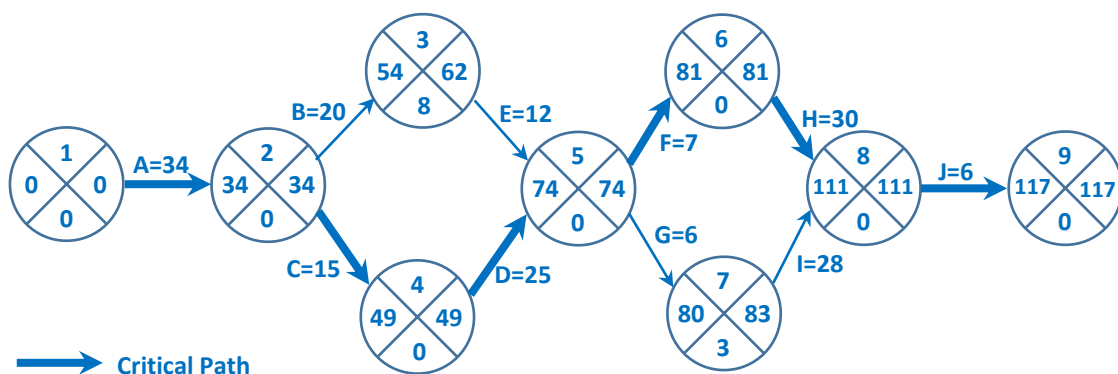
- Critical path: A-E (**longest path** and with a **slack of zero**)
- Project duration 34 weeks
- Other paths and durations: B-D-G-H= 31 weeks  
B-C= 17 weeks  
A-F-H = 19 weeks

## SOLUTION of EXAMPLE 3:

1. The naming convention for **Activity-on-arrow network diagram**

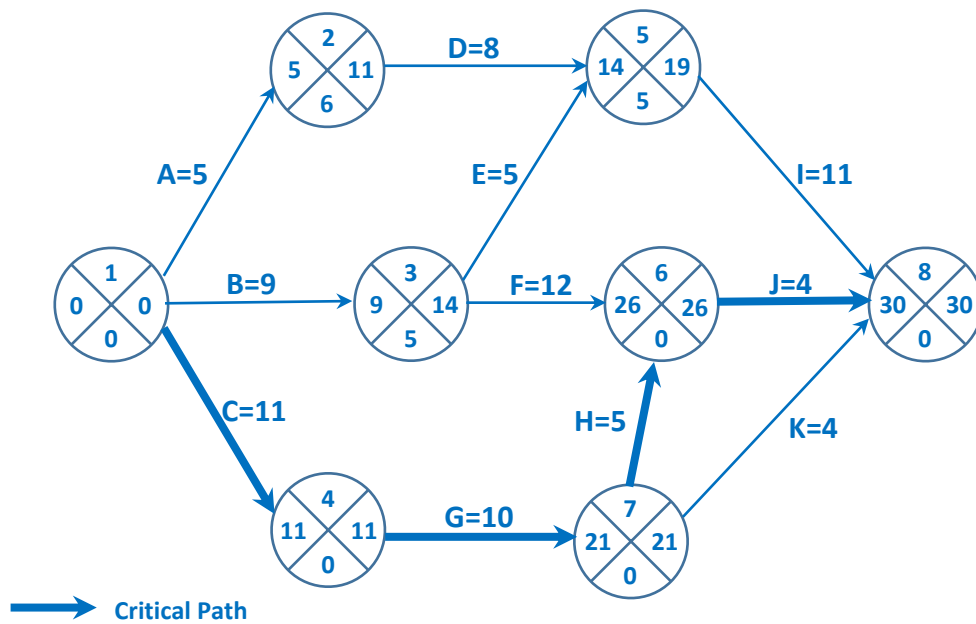


2.



## SOLUTION of EXAMPLE 4:

1.



2. Critical path (= **longest path** and with a **slack of zero**) C-G-H-J = 30 weeks

POSSIBLE PATHS	DURATION
A-D-I	$5 + 8 + 11 = 24$
B-E-I	$9 + 5 + 11 = 25$
B-F-J	$9 + 12 + 4 = 25$
<b>C-G-H-J (Critical path)</b>	<b><math>11 + 10 + 5 + 4 = 30</math></b>
C-G-K	$11 + 10 + 4 = 25$

3. Path **A-D-I** will be the Critical path and duration = 31 weeks